ADAPTIVE PRICE EXPECTATIONS AND THE MODELLING OF WORLD GOLD PRICE DYNAMICS

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Abstract:

Market models that incorporate inventories without including expectation factors assume that all inventories are unintended. This assumption is not true in mineral commodity markets where hedging and speculation are rife. This study investigates how the introduction of price expectations alters the results of an earlier basic market model for gold based only on current price and inventory factors. Expectations-augmented demand and supply functions, together with an inventory formulation, are used to develop a second-order linear difference equation in prices, whose solution is definitized into a price time path by application of 1980-2009 data. Demand and supply functions obtained are largely consistent with theory. The expected-price variable has greater marginal impact coefficients than current price. The introduction of adaptive expectations makes the price time path slightly divergent, and the intertemporal equilibrium price becomes variable as it depends on lagged inventory accumulation and time. Unlike in the earlier study, a retrospective computation of equilibrium prices yields positive values. The model's predictive power does not improve significantly over that of the basic model.

Keywords: Price expectations, inventory, coefficient of expectation, gold market, dynamic stability

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Introduction:

In an earlier study (Mlambo, 2012a) the author used a simple dynamic market model to estimate the time path of gold prices. In that model quantity demanded and quantity supplied were functions of only current prices. An inventory price-relationship was also estimated. The estimated model was insignificant and yielded a negative intertemporal equilibrium price. The model was not useful for prediction purposes. Mlambo (2012b) suggests that hedging and speculation which are rife in the market make consideration of price expectations important.

The general objective of this study is to find out if consideration of adaptive expectations significantly alters the basic conclusions from the earlier simpler study (Mlambo, 2012a) as summarised in the paragraph above. The study develops price expectations-augmented demand and supply functions, which, together with the inventory-price relationship, are used to develop a second-order difference equation in prices. A general solution to this model (in the form of a general price time path) is derived, which is then definitized by application of demand, supply and price data from 1980 to 2009. The dynamic characteristics of the price time path are discussed, in terms of whether there is evidence of converge or divergence. The study also yields estimates of the stock-induced price adjustment coefficient and the coefficient of expectation.

The major structural difference between this study and the simpler one alluded to above is that in this study an indirect distinction is made between unintended and intended inventory flows, while in the earlier study all inventories were assumed to be unintended. Unintended inventories are those which naturally arise from the difference between what is supplied to the market and market sales, while intended inventories are a result of expectations on prices by the producer (that is, hedging) and market investors (speculation). In practice, it is difficult to split inventories into the two categories; however, including price expectations into the demand and supply equations implicitly achieves the desired separation.

Literature review:

Much has been written on the role of price expectations on the demand, supply and actual price dynamics of a commodity. McRae (1978) contends that if resources are taken to be assets, their owners would respond to price expectations. If the price of a natural resource is expected to rise faster than the resource's opportunity cost (in the form of interest), owners of the resource would choose to leave the resource unexploited (or at least reduce their rate of exploitation) in order to make a capital gain later. Expectations tend to be self-fulfilling in that they engender behaviour that produces the anticipated outcome even faster. Mining supply is generally inelastic because of the huge required investments associated with expansion of production and huge fixed costs associated with cutting down operations. Thus, it is often not an easy economic decision either to reduce or increase production. In such a case price expectations directly convert into either decreases or increases in intended inventories.

Expectations tend to be the main driving force behind behaviour in futures, forward contracts, and options markets, including hedging and speculation. Abken (1980) argues that spot prices themselves tend to be influenced by future anticipated prices because, where a good is storable, current flows of supply and demand compare insignificantly to the amount of stocks (the latter are, to a large part, influenced by expected prices through intended inventories). Gold is more liquid than most commodities, and for that reason, is more influenced by future price expectations than by current prices. The two points just made above imply that expected prices relegate current prices to insignificance in determination of both commodity flows and stocks.

Capie *et al* (2004) undertook to test the long-held hypothesis that gold is a hedge against the US$. The theoretical proposition is that, economic agents seek to cover themselves against exchange fluctuations by buying gold when US$ is depreciating, thereby causing the gold price to rise, or selling gold when the US$ is appreciating, thereby causing the gold price to fall. They confirmed statistically the opposite relationship between US$ exchange rate and gold (spot) price, and that it is robust (true in periods of world economic turbulence or stability). Since gold is also traded in futures and forward markets where prices reflect agents' expected prices (which in turn partly depend on expected movements in exchange rates), 'contangos' tend to progressively push up spot prices, while 'backwardations' do the opposite. Arguments of gold as hedge against general inflation are made in Ghosh *et al* (2002), whereby gold price rises at the rate of general inflation.

The desire to predict indicators and outcomes has motivated the study of expectations and intentions data (Muth, 1961. *The rest of this section is based on this reference*). Expectations affect dynamic economic processes; thus, correctly modelling expectation formation processes is important for accurate modelling of the dynamic economic processes. However, the problem has been to understand how expectations are formed. Muth contends that even though various expectation models have been used there has been little evidence that they are consistent with actual economic phenomena.

Apparently, the expectation formation process itself is dynamic, depending as it does on the amount of information available. It may be proposed, in consistence with this argument, that any modelling of expectations or their incorporation into other dynamic models should be sensitive to time-length vis-à-vis changes in information availability. A future research question is: can we produce a "permanent" expectation model into which is built automatic adjustment relative to changes in information availability, so that the model becomes rational when more information is available and adaptive when only historical information is available?

Wrong assumptions on expectations can result in statistically insignificant estimates, and it would be important to check if the assumed expectation formation process would result in actual observed data approximated. Muth generally reports that expectations based models have underestimated actual values. He hypothesizes that dynamic economic models based on rationality do not produce results consistent with observed phenomena because they do not incorporate enough rationality. However, it may be argued that a model that would incorporate all information would be impossible to handle, and that no economic agent can take into account all possible information because: (i) not all information is available to all; and (ii) even if the information was available, the agent would not be able to process all of it into a decision. This suggests that the reality is in fact consistent with the assumption of limited rationality.

Modelling the world gold market:

The presentation of this model is based on Chiang (1984), besides the whole extensions necessitated by the explicit inclusion of price expectations. Two assumptions are made, namely: (a) quantity demanded (Q_{dt}) and quantity supplied (Q_{st}) in period t are linear functions of the current price (P_t) and future price expectations held in period t (P_t^e); and (b) sellers, in setting prices each year, generally take into account their inventories, so as to reduce them by lowering price if they increased, or to increase them by raising the price, if they decreased.

From the above assumptions:

$$Q_{dt} = \beta_0 - \beta_1 P_t + \beta_2 P_t^e + w_t \qquad (1)$$

$$Q_{st} = -\alpha_0 + \alpha_1 P_t - \alpha_2 P_t^e + u_t \qquad (2)$$

$$P_{t+1} = P_t - \theta(Q_{st} - Q_{dt}) \qquad (3)$$

$$\beta_i, \alpha_i, \theta > 0 \qquad (4)$$

Note that the term in parenthesis in the RHS of (3) denotes inventories accumulated in the current period. θ is the *stock-induced price-adjustment coefficient*. Equation (3) shows how inventories affect the price.

Let us assume the following price expectation formation process (equation 5) (See Gujarati, 1988, p.517. *The rest of the derivation of the demand function below is based on this reference*):

$$P_t^e - P_{t-1}^e = \delta(P_t - P_{t-1}^e) \qquad (5)$$

where $0 < \delta \leq 1$. δ is the coefficient of expectation. This formulation is termed *adaptive expectation*, *progressive expectation* or the *error learning hypothesis*. Substituting (5) into the demand function (equation 1) gives:

$$Q_{dt} = \beta_0 - \beta_1 P_t + \beta_2 [\delta P_t + (1-\delta)P_{t-1}^e] + w_t$$

$$\therefore Q_{dt} = \beta_0 - (\beta_1 - \beta_2\delta)P_t + \beta_2(1-\delta)P_{t-1}^e + w_t \qquad (6)$$

Lag equation (1) one period, multiply the result by the factor (1-δ) and subtract the resulting equation from (6):

$$Q_{d,(t-1)} = \beta_0 - \beta_1 P_{t-1} + \beta_2 P_{t-1}^{e} + w_{t-1}$$

$$(1-\delta)Q_{d,(t-1)} = \beta_0(1-\delta) - \beta_1(1-\delta)P_{t-1} + \beta_2(1-\delta)P_{t-1}^{e} + (1-\delta)w_{t-1} \quad (7)$$

$$\therefore Q_{dt} = \beta_0\delta - (\beta_1 - \beta_2\delta)P_t + \beta_1(1-\delta)P_{t-1} + (1-\delta)Q_{d,(t-1)} + w_t - (1-\delta)w_{t-1} \quad (8)$$

$$\therefore Q_{dt} = \phi_1 - \phi_2 P_t + \phi_3 P_{t-1} + \phi_4 Q_{d,(t-1)} + e_t \quad (9)$$

where $\phi_1 = \beta_0\delta$, $\phi_2 = \beta_1 - \beta_2\delta$, $\phi_3 = \beta_1(1-\delta)$, $\phi_4 = 1-\delta$, $e_t = w_t - (1-\delta)w_{t-1}$

Note that once equation (9) is estimated all the parameters in equation (8) can be computed.

By a similar derivation process it can be shown that:

$$Q_{st} = -\alpha_0\delta + (\alpha_1 - \alpha_2\delta)P_t - \alpha_1(1-\delta)P_{t-1} + (1-\delta)Q_{s,(t-1)} + u_t - (1-\delta)u_{t-1} \quad (10)$$

$$\therefore Q_{st} = \phi_5 + \phi_6 P_t + \phi_7 P_{t-1} + \phi_8 Q_{s,(t-1)} + v_t \quad (11)$$

where $\phi_5 = -\alpha_0\delta$, $\phi_6 = \alpha_1 - \alpha_2\delta$, $\phi_7 = -\alpha_1(1-\delta)$, $\phi_8 = 1-\delta$, $v_t = u_t - (1-\delta)u_{t-1}$.

From equation (3):

$$(P_{t+1} - P_t) = -\theta(Q_{st} - Q_{dt})$$

$$P_{t+1}^{d} = \phi_{10} V_t$$

where $P_{t+1}^{d} = P_{t+1} - P_t$, $\phi_{10} = -\theta$ and V_t $\left(= Q_{st} - Q_{dt}\right)$ is inventory accumulation in period *t*. Adding a constant and an error term we get:

$$P_{t+1}^{d} = \phi_9 + \phi_{10} V_t + \mu_t \qquad (12)$$

Equation (12) implies:

$$P_t^{d} = \phi_9 + \phi_{10} V_{t-1} + \mu_{t-1} \qquad (13)$$

Thus, our model now becomes:

$$Q_{dt} = \phi_1 - \phi_2 P_t + \phi_3 P_{t-1} + \phi_4 Q_{d,(t-1)} + e_t \qquad (9)$$

$$Q_{st} = \phi_5 + \phi_6 P_t + \phi_7 P_{t-1} + \phi_8 Q_{s,(t-1)} + v_t \qquad (11)$$

$$P_t^{d} = \phi_9 + \phi_{10} V_{t-1} + \mu_{t-1} \qquad (13)$$

Substituting (9) and (11) into (12) and simplifying gives difference equation (14). Note that since the difference equation is a dynamic equation (showing how price changes over time) we can regard the lagged quantity variables on the RHS of equation (14), *per se*, as parameters.

$$\begin{aligned} P_{t+1} - \left[1 + \phi_{10}\left(\phi_6 + \phi_2\right)\right] P_t - \phi_{10}\left(\phi_7 - \phi_3\right) P_{t-1} &= \phi_9 + \phi_{10}\left(\phi_5 - \phi_1\right) + \phi_{10}\phi_8 Q_{s,t-1} \\ &\quad - \phi_{10}\phi_4 Q_{d,t-1} + \phi_{10}\left(v_t - e_t\right) + \mu_t \end{aligned} \qquad (14)$$

Let $\phi_{11} = -\left[1 + \phi_{10}\left(\phi_6 + \phi_2\right)\right]$, $\phi_{12} = -\phi_{10}\left(\phi_7 - \phi_3\right)$, $\phi_{13} = RHS$ of (14).

Equation (14) is rewritten as:

$$P_{t+1} + \phi_{11}P_t + \phi_{12}P_{t-1} = \phi_{13} \quad (15)$$

Equation (15) is a second-order linear non-homogeneous difference equation with constant coefficients and a constant-term. It can be shown that the particular solution to (15) is of the general form (Chiang, 1984):

$$P_p = \frac{\phi_{13}}{\phi_{11} + 2}t \ , \qquad \text{for } \phi_{11} + \phi_{12} = -1, \quad \phi_{11} \neq -2 \quad (16)$$

It can also be shown that the homogeneous aspect of equation (15) has the characteristic equation given by:

$$\eta^2 + \phi_{11}\eta + \phi_{12} = 0$$

This solves to two characteristic roots:

$$\eta_1, \eta_2 = \frac{-\phi_{11} \pm \sqrt{\phi_{11}^2 - 4\phi_{12}}}{2} \quad (17)$$

Equation (17) in this study gives two distinct real roots as $\left(\phi_{11}^2 > 4\phi_{12}\right)$ (This will be shown later). Thus, the complementary solution is of the form:

$$P_c = A_1\eta_1^t + A_2\eta_2^t \quad (18)$$

Combining the complementary solution with the particular solution we get the price time path:

$$P_t = A_1\eta_1^t + A_2\eta_2^t + \frac{\phi_{13}}{\phi_{11} + 2}t \quad (19)$$

Convergence of the price time path or lack of it will then depend on whether the dominant characteristic root has an absolute value less than 1 (for convergence) or greater than 1 (for divergence). Note that, because of the existence of lagged quantity demanded and supplied variables on the RHS of (14), the intertemporal equilibrium price (which is equal to the last term in the RHS of equation 19) cannot be definitized to a single numerical value.

Equations (9) and (11) are autoregressive, distributed-lag and simultaneous equations. For each of the equations we first deal with the problem of autoregressiveness by the instrumental variable approach. We run the two equations separately by OLS, generate estimated quantity variables (that is, estimated quantity demanded and estimated quantity supplied), and then lag them to get instrumental variables for the lagged demand and lagged supply variable. Thus equation (9) and (11) respectively become:

$$Q_{dt} = \phi_1 - \phi_2 P_t + \phi_3 P_{t-1} + \phi_4 Q^{es}_{d,t-1} + e_t \qquad (20)$$

$$Q_{st} = \phi_5 + \phi_6 P_t + \phi_7 P_{t-1} + \phi_8 Q^{es}_{s,t-1} + v_t \qquad (21)$$

where *es* denotes instrumental variables. Lagged price, lagged inventories, lagged quantity demanded (estimate) and lagged quantity supplied (estimate) are all predetermined variables. Endogenous variables in the model include current price and current quantities.

Using the rank order condition of identification, equations (20) and (21), like their counterparts (9) and (11), are overidentified, while (13) is exclusively a function of a predetermined variable. Thus, the Two Stage Least Squares (2SLS) method is appropriate for estimation of equations (20) and (21) while the Ordinary Least Squares (OLS) method is appropriate for estimation of equation (13). For (20) and (21), as the first stage of the 2SLS, we replace the current price variable on the RHS with an instrumental variable given by:

$$P^{es}_t = \varpi_0 + \varpi_1 V_{t-1} + \varpi_2 Q^{es}_{d,t-1} + \varpi_3 Q^{es}_{s,t-1} + \varpi_4 P_{t-1}$$

where the quantity variables are instrumental variables generated above for the lagged quantity variables (that is, autoregressive variables). However, note that since the estimate price variable is not exactly the same as the price variable the problem of distributed-lag falls away so that we can proceed to apply OLS to (22) and (23) below as the second stage of 2SLS. However, applying an *ad hoc* method of sequential regression for dealing with distributed lags at this stage leads to the same results.

$$Q_{dt} = \phi_1 - \phi_2 P_t^{es} + \phi_3 P_{t-1} + \phi_4 Q_{d,t-1}^{es} + e_t \quad (22)$$

$$Q_{st} = \phi_5 + \phi_6 P_t^{es} + \phi_7 P_{t-1} + \phi_8 Q_{s,t-1}^{es} + v_t \quad (23)$$

<u>Findings and discussion:</u>

Application of the OLS method on equation (22), (23) and (13) using data in Table 1 (second, fourth and fifth column) for 1980-2009 yields the following results:

<u>Secondary expectations-augmented demand function:</u>

Version not corrected for autocorrelation

$$\begin{aligned} Q_{dt} &= -488{,}079.2756 - 109.3801 P_t^{es} + 129.6917 P_{t-1} + 1.1342 Q_{d,t-1}^{es}, \qquad R^2 = 0.84 \\ &\quad (1{,}191{,}828.4) \quad (196.5282) \quad (228.2827) \quad (0.3328) \qquad df = 24 \\ t &= (-0.410) \quad (-0.557) \quad (0.568) \quad (3.408) \qquad F_{3,24} = 41.30 \end{aligned} \quad (24)$$

Version corrected for autocorrelation by Cochrane-Orcutt method

$$\begin{aligned} Q_{dt} &= 2{,}425{,}152.91 - 211.4954 P_t^{es} + 241.3908 P_{t-1} + 0.3409 Q_{d,t-1}^{es}, \\ &\quad (1{,}407{,}015.1) \quad (177.7784) \quad (202.6540) \quad (0.3644) \\ t &= (1.724) \quad (-1.190) \quad (1.191) \quad (0.936) \end{aligned} \quad (25)$$

The uncorrected version of the secondary demand function is consistent with demand theory except in the negative constant and the positive coefficient of lagged demand. The explanatory ability of the model is very high at 84% and the model as a whole is very significant with a calculated F value of 41.30 against the critical value of 3.01 at 5% level of significance. However, with the critical t-values of ±2.064, both current and lagged price coefficients are insignificant while the lagged quantity demanded variable is significant.

Since the data are time-series, in equation (25) we present results corrected for autocorrelation (which is likely to arise from the data series). The results improve greatly as the constant is now positive and the price coefficients have the right signs and have improved in significance (though they are still insignificant). However, the lagged demand variable is now also insignificant.

<u>Secondary expectations-augmented supply function:</u>

Version not corrected for autocorrelation

$$
\begin{aligned}
Q_{st} &= 1{,}031{,}372.55 + 179.7935 P_t^{es} - 202.8487 P_{t-1} + 0.7192 Q_{s,t-1}^{es}, \quad R^2 = 0.91 \\
&\quad (426{,}571.41) \quad (76.1459) \quad (89.1698) \quad (0.1170) \quad df = 24 \qquad (26) \\
t &= (2.418) \quad (2.361) \quad (-2.275) \quad (6.147) \quad F_{3,24} = 82.34
\end{aligned}
$$

Version corrected for autocorrelation using Cochrane-Orcutt method

$$
\begin{aligned}
Q_{st} &= 1{,}916{,}356.91 + 275.2757 P_t^{es} - 298.8359 P_{t-1} + 0.3985 Q_{s,t-1}^{es}, \\
&\quad (470{,}551.16) \quad (81.2034) \quad (96.0284) \quad (0.1511) \qquad (27) \\
t &= (4.073) \quad (3.390) \quad (-3.112) \quad (2.637)
\end{aligned}
$$

In both the corrected and the uncorrected results price coefficients (current and lagged) are consistent with theoretical expectations. The results show a very high explanatory ability at 91% and the model as a whole is significant with an F value of 82.34. All coefficients are significant at 5% in both the uncorrected and corrected versions. However, a positive constant is unexpected.

Inventory function:

Version not corrected for autocorrelation

$$P_t^d = 545.6830 - 0.0011V_{t-1} \qquad R^2 = 0.02$$
$$(404.8488) \quad (0.0015) \qquad df = 26 \qquad (28)$$
$$t = (1.348) \quad (-0.747) \qquad F_{1,26} = 0.56$$

Version corrected for autocorrelation using the Cochrane-Orcutt method

$$P_t^d = 895.74 - 0.0008V_{t-1}$$
$$(689.9077) \quad (0.0013) \qquad (29)$$
$$t = (1.298) \quad (-0.621)$$

Qualitative results on the inventory function (both the uncorrected and the corrected versions) are consistent with theoretical expectations. However, the results are insignificant, with computed t in both versions (= -0.747 and -0.621 respectively) > -2.056, computed F-value < 4.23. The explanatory power is also very low at 2%.

From the above results (corrected versions) we get the following values of the secondary parameters (all subsequent interpretations and discussions are based on corrected results):

$$\phi_1 = 2{,}425{,}152.91, \quad \phi_2 = 211.4954, \quad \phi_3 = 241.3908, \quad \phi_4 = 0.3409, \quad \phi_5 = 1{,}916{,}356.91,$$
$$\phi_6 = 275.2757, \quad \phi_7 = -298.8359, \quad \phi_8 = 0.3985, \quad \phi_9 = 895.74, \quad \phi_{10} = -0.0008,$$
$$\phi_{11} = -0.6106, \quad \phi_{12} = -0.4322$$

The stock-induced adjustment coefficient and the coefficient of expectation:

The stock-induced price adjustment coefficient, given by $-\phi_{10}$ is 0.0008 or 0.08%. Thus, if there is inventory accumulation (positive flow) in the current year, next year's price will be lower than the current price by 0.08% of the current year inventory accumulation and the converse is true. This is consistent with theoretical expectations which postulate a positive value for θ.

There are two values of the coefficient of expectation (δ), emanating from $\phi_4 = 1 - \delta$ (denoted by δ_d, from the demand-side) and $\phi_8 = 1 - \delta$ (denoted by δ_s, from the supply-side). The respective estimates are 0.6591 (= 65.91%) and 0.6015 (= 60.15%), showing that suppliers' reaction, in terms of price expectation adjustments, to expectation error is slightly lower in magnitude than that of buyers. This means that, while theoretically, the expectation formation processes for buyers and sellers are normally assumed to be identical, empirically, because of information asymmetry between the two groups, they differ quantitatively.

The primary demand function, supply function and the inventory-price relationship:

Note that:

$$\beta_0 = \frac{\phi_1}{\delta_d},\ \beta_1 = \frac{\phi_3}{\phi_4},\ \beta_2 = \frac{\left(\phi_3/\phi_4\right) - \phi_2}{\delta_d},\ \alpha_0 = -\frac{\phi_5}{\delta_s},\ \alpha_1 = -\frac{\phi_7}{\phi_8},\ \alpha_2 = \frac{\left(-\phi_7/\phi_8\right) - \phi_6}{\delta_s}$$

Thus, we obtain the primary demand function (equation 1), the primary supply function (equation 2), and the primary inventory-price relationship (equation 3) as:

$$Q_{dt} = 3{,}679{,}491.59 - 708.0986P_t + 753.4565P_t^e \quad (30)$$

$$Q_{st} = 3{,}185{,}963.28 + 749.9019P_t - 789.0710P_t^e \quad (31)$$

$$P_{t+1} = P_t - 0.0008\left(Q_{st} - Q_{dt}\right) \quad (32)$$

The estimates of the demand function (that is, the constant, current price coefficient and expected price coefficient) are all consistent with theoretical expectations. The demand constant shows that when both current and expected prices are zero world demand would be about 3.7 million kilograms. The demand results also indicate that when the current annual price increases by one unit, annual quantity demanded would fall by 708 kilograms, while a marginal increase in expected price would increase current demand by 753 kilograms. This summarily shows that price expectations have a greater impact on demand than current prices.

In the supply function for gold the coefficients of both current price and expected price are consistent with theoretical expectations. A unit increase in current price increases supply by 750 kilograms, while a similar increase in the expected price reduces supply by 789 kilograms. This, like in the demand case, shows that suppliers of gold respond more to changes in price expectations than to changes in current prices, a conclusion also reached by Abken (1980). The positive constant in the supply function is inconsistent with theory, which postitulates a negative supply intercept. However, this shows that there are other non-price factors in the world market of gold that influence supply. Mlambo (2012b) delves into these other factors in detail.

The inventory-price relationship has been discussed under the stock-induced price adjustment coefficient estimate.

The price time path:

Equation (15), ignoring the error terms, can be definitized to:

$$P_{t+1} - 0.6106 P_t - 0.4322 P_{t-1} = 1{,}302.7768 - 0.0003 Q_{s,t-1} + 0.0003 Q_{d,t-1}$$

That is:

$$P_{t+1} - 0.6106 P_t - 0.4322 P_{t-1} = 1{,}302.7768 - 0.0003 V_{t-1} \qquad (33)$$

The sum of the coefficients of current and lagged prices in (33) is -1.04282, which is not significantly different from -1 (it is equal to -1 in whole number). Thus, we may use the particular solution given by equation (16). That is:

$$P_p = \frac{1{,}302.7768 - 0.0003V_{t-1}}{1.3894}t = \left(937.6542 - 0.0002V_{t-1}\right)t \quad (34)$$

The characteristic roots of equation (33) are given by:

$$\eta_1, \eta_2 = \frac{0.6106 \pm \sqrt{(-0.6106)^2 - 4(-0.4322)}}{2}$$

$$\eta_1 = 1.0302, \quad \eta_2 = -0.4196$$

This is therefore a case of real distinct roots. In this case the complementary function is given by the general function:

$$P_c = A_1\eta_1^{\,t} + A_2\eta_2^{\,t} \quad (18)$$

That is:

$$P_c = A_1(1.0302)^t + A_2(-0.4196)^t \quad (35)$$

The complete solution becomes:

$$P_t = A_1(1.0302)^t + A_2(-0.4196)^t + \left(937.6542 - 0.0002V_{t-1}\right)t \quad (36)$$

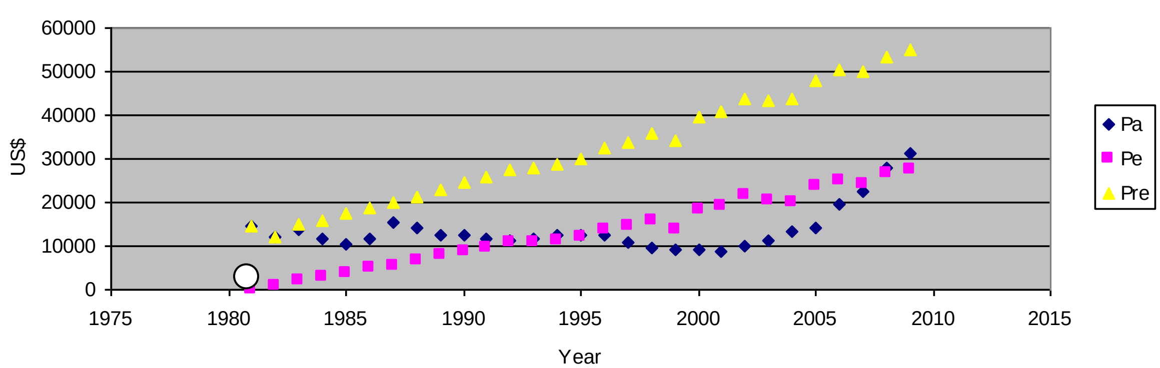
Taking prices for 1981 and 1982 as initial conditions (since the lagged variable has no value for 1980 in the data set used) equation (36) is definitized to:

$$P_t = 11{,}942.8926(1.0302)^t + 2{,}834.8574(-0.4196)^t + \left(937.6542 - 0.0002V_{t-1}\right)t \quad (37)$$

The equilibrium price, given by $(937.6542 - 0.0002V_{t-1})t$, is not a single value since it varies with lagged inventory accumulation and time – it is a moving equilibrium. All equilibrium values computed over the period are positive unlike in the basic model where there is a single negative value. In Table 1 in the appendix the series of equilibrium prices for 1982 – 2009 is given in the second last column. A retrospective prediction of prices using equation (37) gives the last column in Table 1.

Figure 1 below shows the trends for actual, predicted and intertemporal equilibrium prices in one plane. Note that the trend for equilibrium price in Figure 1 is beginning at zero for 1981 simply because we assign a t = 0 for that year (t enters multiplicatively into the equilibrium formula). Therefore, for 1981 no prediction is in fact given. In the figure we insert a white circle over the scatter point on 1981 to signify 'no prediction'.

Figure 1: Trends of Actual (Pa), Intertemporal Equilibrium (Pe) and Predicted (Pre) Prices for Gold



A number of interesting observations can be made from these results. Since the dominant characteristic root (= 1.0302) is slightly greater than 1 in absolute value the price time path (yellow) slightly diverges without oscillations from the intertemporal equilibrium trend (pink). This differs from the basic model in which the path was convergent. From Figure 1 it is clear that, except for two years (1981 and 1982), the model overestimated actual prices. Thus, it is not a significant improvement over the basic model in terms of predictive ability. Actual prices (blue trend) fluctuated around the equilibrium trend.

Conclusion:

The study shows that incorporation of adaptive price expectations results in demand, supply and inventory functions that are largely normal. However, the coefficients of current price, lagged price and lagged quantity demanded in the secondary demand function are individually insignificant, though jointly significant. Price expectations have a greater impact on current demand and supply of gold than current prices. The inclusion of expectations changes the dynamic characteristics of the gold price time path in at least three ways. It makes the price time path slightly divergent. The intertemporal equilibrium price is not a single value, but varies with the lagged value of inventory accumulation and time. Thus, not only is the equilibrium dynamically unstable, but the equilibrium price itself is also unstable. Unlike the simple model which obtained a negative equilibrium price, in all instances the equilibrium prices computed are positive. Also, unlike the basic model which underestimates actual prices, this model overestimates. Thus, the current model does not essentially represent a significant improvement in predictive performance. There is need to investigate whether or not a change in expectation formation assumption and/or an inclusion of global income as a factor of demand would not make the model better, a recommended question for further research.

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Appendix:

Table 1: Global Gold Statistics and Model Price Results

Year	World Annual Price (US$/kg)	World Mine Production (kg of metal Content)	World supply of scrap (kgs)	Total World Supply (production + scrap) (kgs)	World Demand (kgs)	Equilibrium Prices	Predicted Prices
1980	19,694.23	1,220,000	500,000	1,720,000	1,617,245		
1981	14,777.75	1,260,000	240,000	1,500,000	1,670,270	-	14,777.75
1982	12,085.77	1,300,000	240,000	1,540,000	1,723,294	971.71	12,085.77
1983	13,631.90	1,400,000	300,000	1,700,000	1,855,855	1,948.63	15,122.88
1984	11,595.47	1,400,000	300,000	1,700,000	1,855,855	2,906.48	15,754.97
1985	10,212.99	1,500,000	340,000	1,840,000	1,988,417	3,875.30	17,415.45
1986	11,839.17	1,564,000	520,000	2,084,000	2,073,256	4,836.69	18,658.35
1987	15,366.43	1,620,000	460,000	2,080,000	2,147,490	5,613.03	19,905.56
1988	14,091.97	1,798,000	380,000	2,178,000	2,383,449	6,658.07	21,359.80
1989	12,300.21	1,942,000	380,000	2,322,000	2,574,337	7,829.95	22,985.09
1990	12,375.77	2,133,000	500,000	2,633,000	2,827,528	8,893.09	24,501.97
1991	11,680.03	2,132,000	460,000	2,592,000	2,826,203	9,765.60	25,847.52
1992	11,091.03	2,233,000	480,000	2,713,000	2,519,100	10,829.44	27,396.34
1993	11,603.51	2,253,000	580,000	2,833,000	2,475,700	10,786.49	27,854.00
1994	12,391.20	2,220,000	620,000	2,840,000	2,460,100	11,260.52	28,843.35
1995	12,394.09	2,160,000	620,000	2,780,000	2,726,000	12,063.44	30,177.32
1996	12,509.19	2,270,000	640,000	2,910,000	2,779,500	13,902.81	32,563.71
1997	10,686.25	2,420,000	620,000	3,040,000	3,053,600	14,584.87	33,809.33
1998	9,488.96	2,460,000	1,080,000	3,540,000	2,714,100	15,986.36	35,791.40
1999	9,002.20	2,520,000	620,000	3,140,000	3,284,200	13,904.54	34,307.69
2000	9,002.20	2,550,000	619,000	3,169,000	3,264,400	18,363.39	39,382.72
2001	8,744.99	2,540,000	749,000	3,289,000	3,729,000	19,134.68	40,788.80
2002	9,998.87	2,530,000	873,500	3,403,500	3,363,000	21,538.74	43,846.81
2003	11,252.75	2,530,000	991,000	3,521,000	3,207,000	20,450.19	43,431.97
2004	13,213.94	2,400,000	881,400	3,281,400	3,515,000	20,121.65	43,797.47
2005	14,339.21	2,510,000	902,400	3,412,400	3,753,000	23,624.98	48,015.81
2006	19,611.93	2,360,000	1,132,800	3,492,800	3,435,000	25,144.36	50,271.79
2007	22,473.34	2,340,000	981,800	3,321,800	3,571,000	24,078.45	49,964.73
2008	28,099.71	2,290,000	1,315,600	3,605,600	3,812,000	26,662.34	53,330.39
2009	31,346.93	2,460,000	1,694,700	4,154,700	3,493,000	27,410.16	54,883.58

Sources of original data and notes: All mine production figures are obtained from Lofty, Sharp, Hillier, Singh, Lehall, Jones, & Davies (1983), Lofty, Sharp, Hillier, & Joseph (1987), Lofty, Hillier, Cooke, Linley, & Singh (1992), Taylor, Lofty, Hillier, Fellows, Bate, Linley, Mills, & White (1995), Taylor, Hillier, Mills, & White (2004), Stockwell, Hillier, Mills, & White (2000), Hetherington, Brown, Benham, Bide, Lusty, Hards, Hannis, & Idoine (2008), and Brown, Bide, Walters, Idoine, Shaw, Hannis, Lusty, & Kendall (2011); 1980-1998 prices are from Amey (2011) and the rest (1999-2009) are from Amey (2001, 2004) and George (2007, 2010, 2011); All prices are converted from dollars per troy ounce (1980-1998) or per ounce (1999-2009) to dollars per kg by multiplying by 32.1507 as indicated in Amey (2011); World demand figures for 1992-2009 are from World Gold Council (1997, 2002, 2011). Demand figures for 1980-1991 are estimated by multiplying the respective production figures by the average ratio of production to demand over the period 1992-2009, equal to 1:1.325611. The source for demand figures for 1996-2000 (World Gold Council, 2002) indicates these are a total of demand in key markets in the world including India, Greater China, Japan, S.Korea, SE Asia, Saudi Arabia, Gulf States, Egypt, Turkey, America and Europe. World supply of scrap figures for 2001-2009 are from GFMS Limited, 2011, p.68, and these are supply from fabricated old scrap. World supply of scrap figure for 2000 is from Grendon International Research Pty Ltd (2010, p.1). World scrap supply figures for 1980-1999 are estimates measured out of the bar chart in Klapwijk (2010).